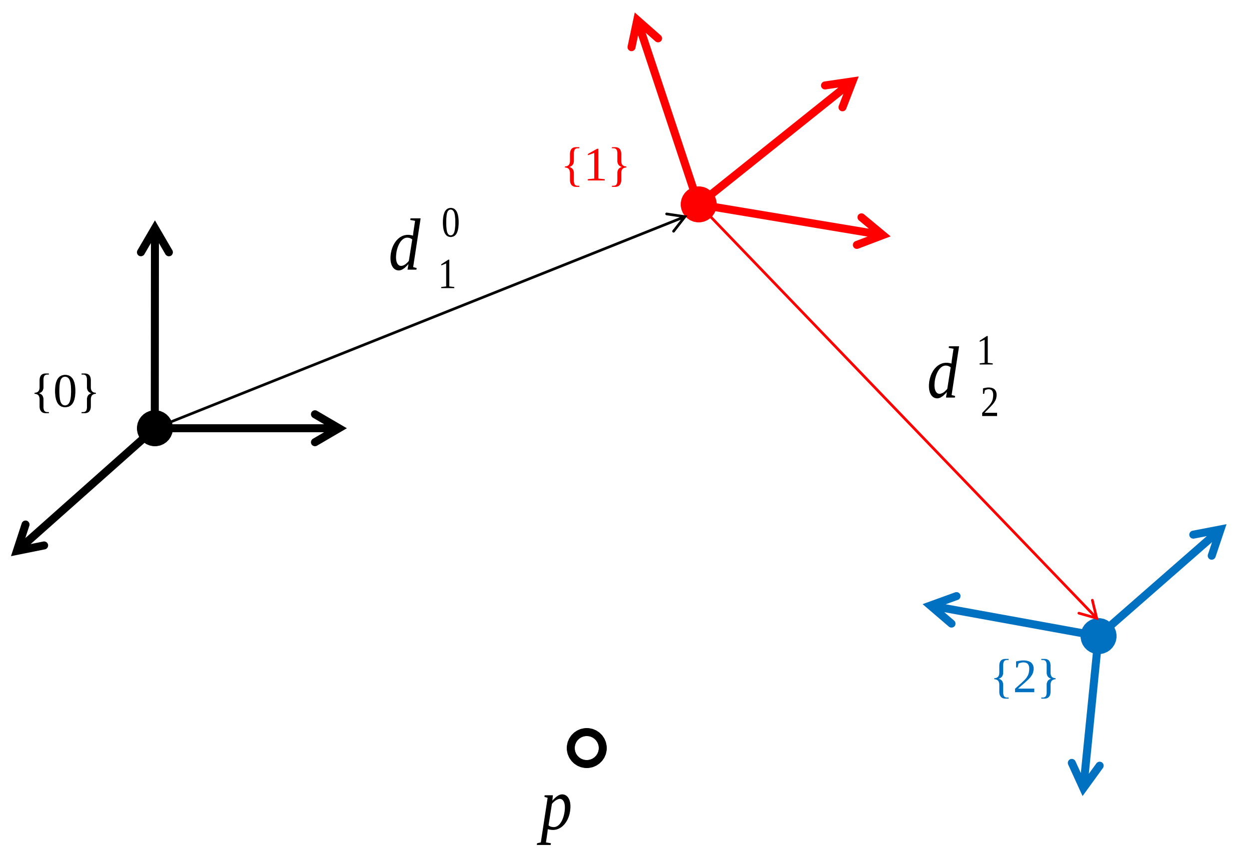
Day 04

Rigid Body Transformations

Rigid Body Transformations in 3D



Homogeneous Representation

- every rigid-body transformation can be represented as a rotation followed by a translation *in the same frame*
  - as a 4x4 matrix

$$T = \begin{bmatrix} & R & & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where R is a 3x3 rotation matrix and d is a 3x1 translation vector

Homogeneous Representation

- in some frame i
  - points

$$P^i = \begin{bmatrix} p^i \\ 1 \end{bmatrix}$$

  - vectors

$$V^i = \begin{bmatrix} v^i \\ 1 \end{bmatrix}$$

Inverse Transformation

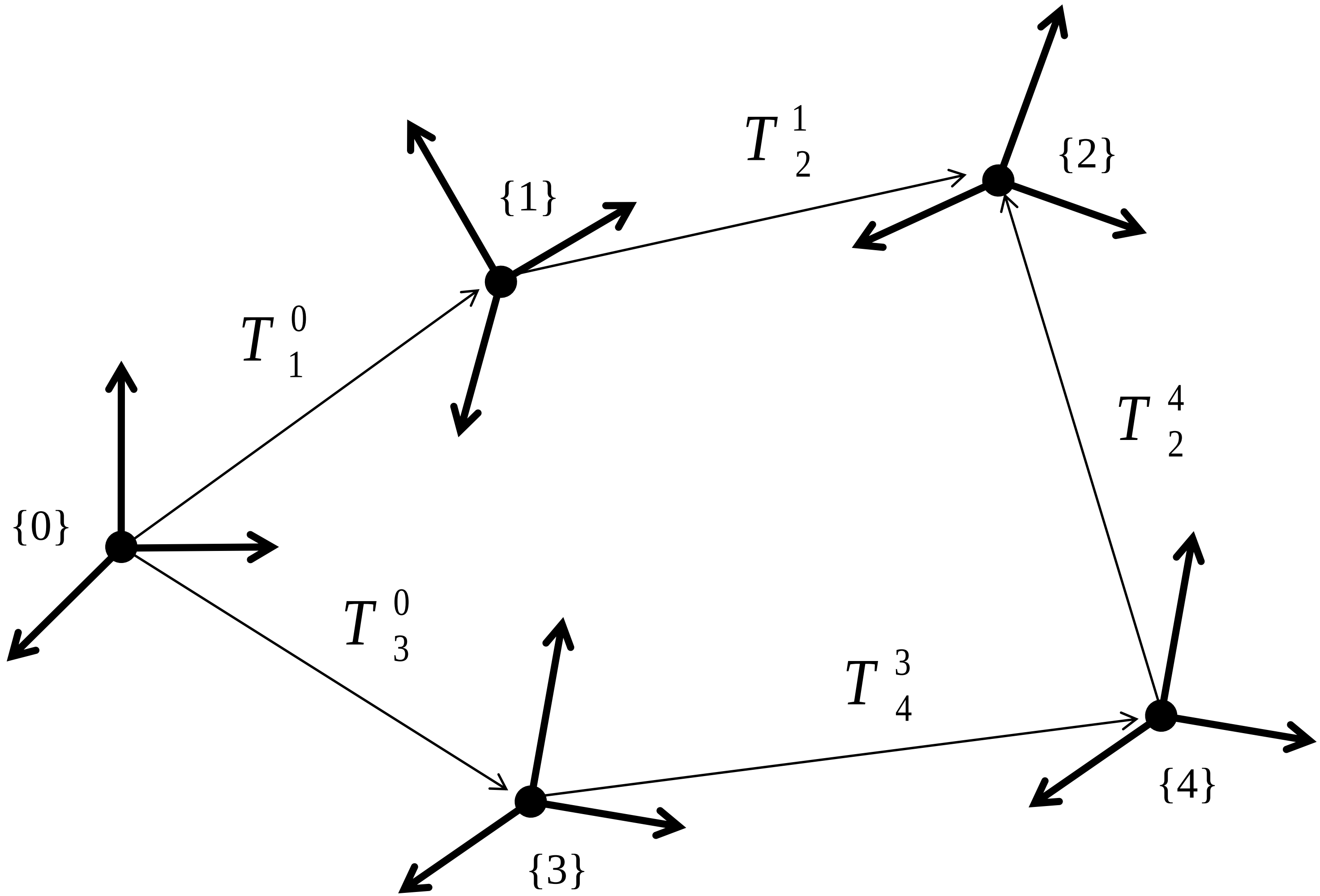
- the inverse of a transformation undoes the original transformation
  - if

$$T = \begin{bmatrix} & R & & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

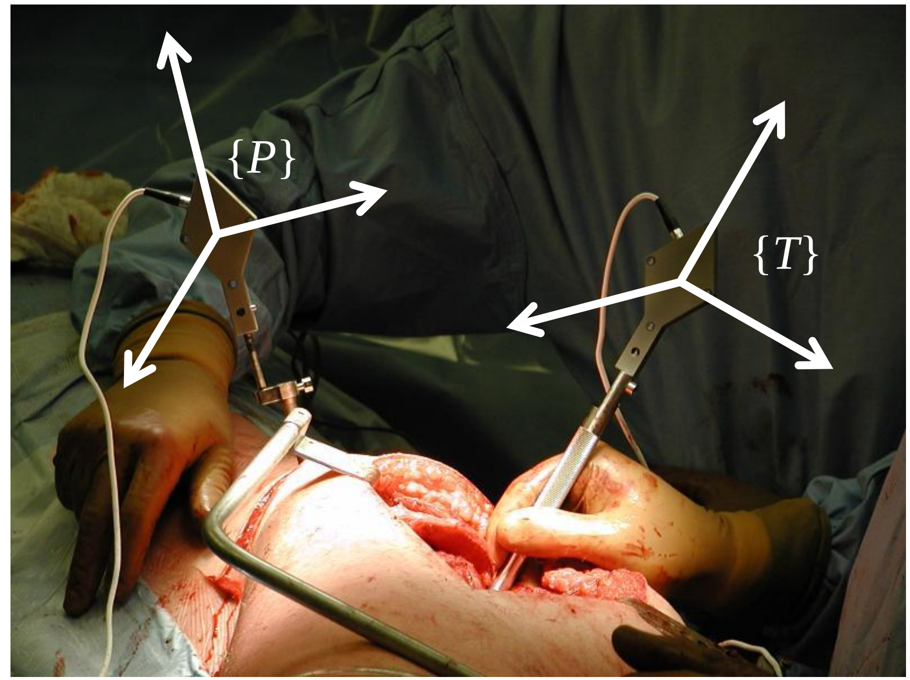
  - then

$$T^{-1} = \begin{bmatrix} & R^T & & -R^T d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

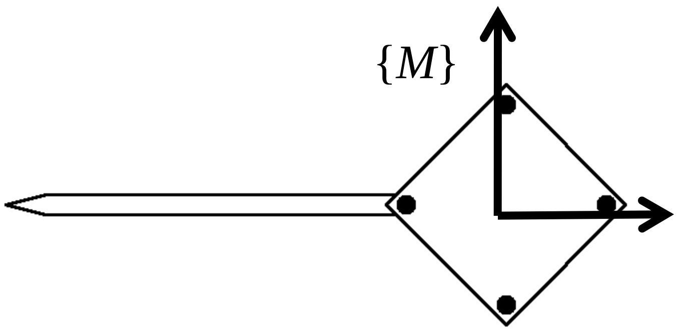
Transform Equations

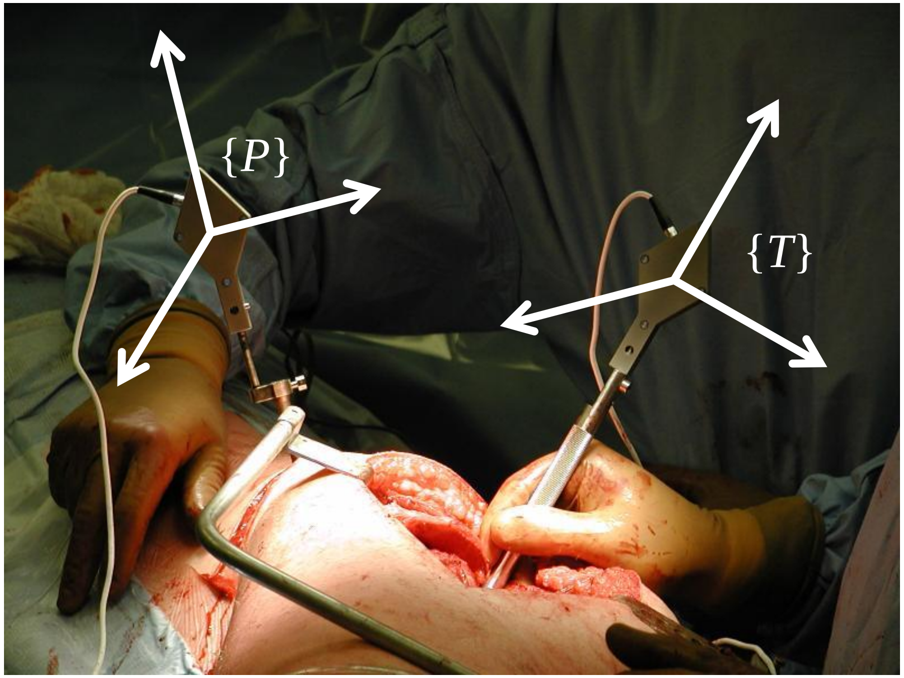


Frames in Computer-Aided Surgery



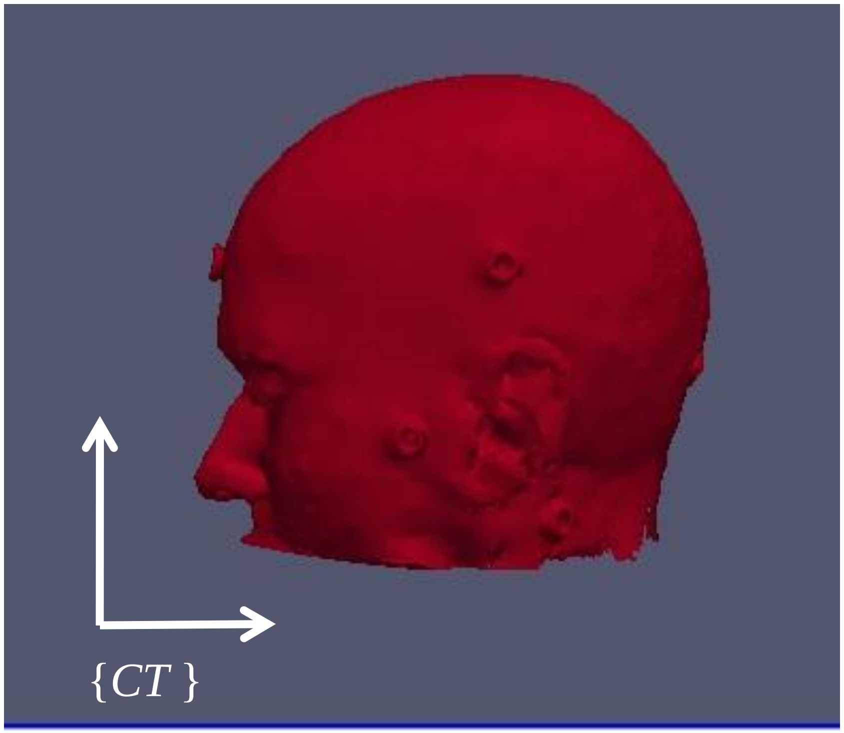
Frames in Computer-Aided Surgery





Frames in CGI

Frames in Computer-Aided Surgery



Fiducial Registration

- physical markers easily locatable in the medical imagery and on the patient are often used in neurosurgery to establish the spatial relationship between the patient and the images
- same fundamental problem appears in many different fields and goes by many different names
  - fiducial registration
  - point-based registration
  - paired-point registration
  - registration with correspondances
  - absolute orientation
  - point fitting
  - …

Problem Statement

- given n points measured in $\{L\}$ and $\{R\}$ estimate the transformation T_L^R (or T_R^L)